Excerpts from "Intelligent Design and Evolutionary Computation"

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> We ought, it seems to me, to consider it likely that the formation of elementary living organisms, and the evolution of those organisms, are also governed by elementary properties of matter that we do not understand perfectly but whose existence we ought nevertheless admit.
>
> Émile Borel, *Probability and Certainty*

1 Introduction

In the United States, a succession of lost legal battles forced opponents of public education in evolution to downgrade their goals repeatedly. By the 1980's, evolution was ensconced in the biology curricula of public schools, and references to the creator of life were illegal. The question of the day was whether instruction in creation, without reference to the creator, as an alternative explanation of life violated the constitutional separation of church and state. In 1987, the U.S. Supreme Court decided that it did, and intelligent design (ID) rose from the ashes of creation science. ID may be seen as a downgraded form of creation. While the creation science movement sought to have biology students introduced to the notion that *creation* is evident in the complexity of living things, the ID movement sought to have students introduced to the notion that *design, intelligence*, and *purpose* are evident.[3] ID preserves everything in the notion of *creation* but the *making*.

[3] The ID movement, led by the Discovery Institute, has downgraded its goals, and presently does not advocate teaching ID in public schools. The Discovery Institute continues, however, to advocate teaching the shortcomings of evolutionary theory.

Although intellectual endeavor is secondary to sociopolitical action in the ID movement, the objective here is to assess the intellectual component. Separating the two is not always possible. Sometimes ID advocates formulate their ideas in ways that make sense only in light of their sociopolitical objectives. The main intellectual offering of ID is the *design inference*, an ostensibly scientific adaptation of the classical argument from design. While the classical argument might indicate that a natural entity is too complex to have arisen unless created by an intelligent and purposive agent, and that the agent could only be God, a design inference eschews creation and declines to identify the agent, concluding that a non-natural and purposive intelligence designed the natural entity.

The sociopolitical ingenuity of the design inference is that, if taught as an alternative to evolution in public-school science classes, it would leave identification of the designer to schoolchildren. The faithful would conclude that science supports belief in God the Designer and disbelief in evolutionary theory. Whether ID's evasion of direct reference to God eventually will pass judicial scrutiny in the U.S. is unknown. The design inference has a legal vulnerability arising from the fact that *non-natural* intelligence is *supernatural*, and the supernatural is clearly linked with the religious in case law [10, p. 67]. In recent years, the ID movement has shifted to saying that intelligence is natural, but not material. (For an example of earlier usage, see [11].) Given that scientists conventionally regard nature to be material, the ID movement has changed the meaning of *natural* to suit itself, and for no apparent reason but to gain better legal footing. Similarly, many ID advocates call themselves evolutionists, falling back on a dictionary meaning of the term (a process of change in a given direction), rather than scientists' conventional interpretation (an undirected process of change deriving from random variation of offspring and natural selection). This chapter will use conventional scientific terminology.

There are two basic approaches to design inference: the argument from *irreducible complexity*, and the argument from *specified complexity*. The argument from irreducible complexity is a demonstration that a biological system with several or more parts could not serve any useful function if any of its parts were removed. This in effect shows that the system cannot have evolved directly. The argument from specified complexity demonstrates that an entity matches some pattern recognized by an intelligent agent, and that it is improbable that any match of the pattern would arise by natural (materialistic) causes in the history of the universe.

The ID movement has for some years considered evolutionary computation (EC) a threat. EC is the proving ground of ideas in evolutionary theory, according to William A. Dembski, billed by the ID movement as the "Isaac Newton of Information Theory" [4]. Positive results in EC apparently contradict Dembski's claims about "conservation of information" in chance-and-necessity

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It bears mention also that the ID movement now distinguishes biological ID and cosmological ID. Here we focus on biological ID.

processes. He and other ID advocates acknowledge this, and are at pains to show that the contradiction is only apparent [4]. Thus EC investigators have a potent means of challenging ID. In fact, the main biological claims of ID advocates such as Michael Behe [1] and Steven Meyer [15] are that evolution cannot account for certain biological innovations that occurred hundreds of millions of years ago, and it is easier to challenge the information-theoretic claims of Dembski with simulation and analysis than it is to challenge the specific biological claims of other ID advocates with new data on ancient events.

Research in artificial life is closely related to that in EC, and in the present context the artificial life program Avida will be considered an example of EC. Research with Avida has been reported in *Nature* [12], and it apparently contradicts the central claim Behe makes in *Darwin's Black Box* [1], namely that gradual evolutionary processes cannot generate irreducible complexity (defined below). In a brief intended for submission to a federal judge, Dembski does not deny that Avida generated irreducible complexity, but instead argues that it lacks biological relevance [6, p. 19].

2 Historical Background

[This section is omitted.]

3 What Is Intelligent Design?

Advocates of ID use the term *intelligent design* to name both their field of inquiry and a putative cause of certain natural phenomena. They refer to their body of beliefs as *intelligent design theory*. Note that the sense of *theory* here is not *scientific theory*.

In ID theory, information is a physical primitive, like matter and energy, which may enter the natural universe from without.[4] An *intelligence* is a non-natural source of information – i.e., it changes probabilities of events in the natural universe. When an intelligence increases the probability of an event that is in some sense meaningful or functional, it is *goal-directed* or *telic*. The central thesis of ID is that some natural entities exhibit such complex organization that the processes giving rise to them cannot have been entirely natural, but instead must have been directed (informed) to some degree by telic intelligence. The type of organization of interest to ID theorists is known as *specified complexity* (or *complex specified information*). An entity with specified complexity higher than ID theory says could have arisen by purely natural processes is said to be *intelligently designed*. ID theorists consider irreducible complexity as an indicator of high specified complexity.

[4] Recall that ID advocates say that intelligence is natural but not material, and that mainstream science holds that anything natural is material.

ID theory says something outside of nature may cause an event within nature. In contrast, mainstream scientists embrace *methodological naturalism*, the working assumption that all natural phenomena have natural causes. ID theory allows theists to associate the intelligent causation of humans with that of one or more deities. In particular, the Biblical notion that humans, as intelligent entities with free will, are created in the image of God, and thus stand apart from nature in some aspects, is supported by the philosophy of intelligent design.

Many ID sites on the Internet (e.g., [9]) offer the definition, attributed to Dembski, "Intelligent Design is the study of patterns in nature that are best explained as the result of intelligence." Although this is a casual definition, its shortcomings are worth examining. First, ID theory permits an event to be explained in terms of both non-natural intelligence and natural antecedents. Design is not all-or-nothing. Second, ID does not study the patterns per se, but *which* patterns indicate that intelligence has contributed information.

The following sections discuss the two main approaches to design inference: *argument from irreducible complexity* and *argument from specified complexity*.

4 Irreducible Complexity

[This section is omitted.]

5 Specified Complexity

For centuries, design advocates, though not the present-day ID advocates, have advanced the *argument from improbability* [16]. The approach is to show some event in nature is very unlikely to have occurred by chance, and to therefore conclude God caused it. But this argument is fallacious. For one thing, low probability does not necessarily justify rejection of chance. When numbers are drawn in a lottery, for instance, it is certain that the chance outcome will be one that was highly improbable *a priori* [16]. Another problem with the argument is that assigning an identity to the cause of the event is unwarranted [5].

William Dembski has developed an analogous *argument from specified complexity*, which concludes that a natural event reflects the intervention of intelligence [3, 4, 7]. That is, a natural event contains information that was introduced purposely by an unidentified source outside of nature. This statement is not quite the same thing as saying intelligence caused the event, because the event may have resulted from a combination of natural causes and intelligent intervention. For instance, one may argue that natural evolutionary mechanisms, while operating as claimed by mainstream scientists, do not account fully for life on earth, and that intelligence has guided (added

information to) evolutionary processes. Note that information may have entered continually, and that there may have never have been a discrete design event.

To conclude that a natural event reflects intelligent design, one must demonstrate that the event is improbable. Dembski describes improbable events as *complex*. One must also demonstrate that the event is *specified* in the sense that it exhibits a pattern that exists independently of itself [3, 4]. To be more precise, there must exist some "semiotic agent" that describes the event with a sequence of signs [7]. Specified complexity, or *complex specified information* (CSI), is a quantity that "factors in" the improbability of the event and the cost of describing it. Dembski claims that when the CSI of an event exceeds a threshold value, inference that intelligence contributed to the event is warranted.

5.1 Design Inference as Statistical Hypothesis Testing

In unpublished work [7], Dembski has reduced the argument from specified complexity [3, 4] to statistical hypothesis testing. The approach is derived from that of Fisher, with a null (or *chance*) hypothesis possibly rejected in favor of an alternative hypothesis. The chance hypothesis says natural causes account entirely for an event in nature, and the alternative hypothesis says the event reflects design (contains information that could only have come from without nature). The Fisherian approach requires specification of the rejection region prior to sampling. But the argument from specified complexity entails selection of a past event and subsequent definition of a rejection region in terms of a pattern found in the event. Dembski claims to have corrected for "data dredging" and *a posteriori* fitting of the rejection region to the event by including *replicational resources* and *specificational resources* as penalty factors in a test statistic [7].

Dembski's argument goes something like this. Suppose $\mathbf{H}$ is the chance hypothesis, and let E be an event in the sample space of $\mathbf{H}$. For any pattern describable by semiotic agent S, there is a corresponding event T containing all matches of the pattern. Dembski uses T to denote both the event and the pattern [7]. The probability (under the chance hypothesis) of matching the pattern is

$$\mathbf{P}(T \mid \mathbf{H}).$$

T serves as a rejection region, and it is possible to make the probability low enough to ensure rejection by choosing a very specific pattern that matches few events, or perhaps no event but E. A penalty factor counters such a "rigged" selection of the pattern.

The *specificational resources* used by S in identifying pattern T are

$$\varphi_S(T),$$

which gives the rank-complexity of the semiotic agent's description of the pattern. In essence, the agent enumerates its pattern descriptions from less complex (e.g., shorter) to more complex (longer), looking for matches of E. The rank-complexity is the least index of a description of pattern T in the enumeration. It is a count of how many descriptions the agent processed to obtain the description of matching pattern T. Dembski [7] considers

$$\varphi_S(T) \cdot \mathbf{P}(T \mid \mathbf{H})$$

"an upper bound on the probability (with respect to the chance hypothesis H) for the chance occurrence of an event that matches any pattern whose descriptive complexity is no more than T and whose probability is no more than $\mathbf{P}(T \mid \mathbf{H})$" for a fixed agent S and a fixed event E. The negative logarithm of this quantity,

$$\sigma = -\log_2[\varphi_S(T) \cdot \mathbf{P}(T \mid \mathbf{H})] \text{ bits},$$

is *specificity*, a type of information [7]. As the probability of matching the pattern goes down, specificity goes up. As the number of patterns "dredged" by the semiotic agent goes up, specificity goes down. Maximizing specificity – and ultimately inferring design in event E – is a matter of finding in the event a simple pattern that is matched with low probability under the chance hypothesis.

Not only is the pattern chosen to obtain high specificity, but the event E and the semiotic agent S, and another penalty is required. The number of *replicational resources* is bounded above by the product of the number of semiotic agents available and the number of events that might have been considered. In applications to biology, Dembski uses as an upper bound Seth Lloyd's estimate of the number of elementary logical operations in the history of the universe, 10^{120} [14]. Dembski claims that if

$$10^{120} \cdot \varphi_S(T) \cdot \mathbf{P}(T \mid \mathbf{H}) < 0.5,$$

then the chance hypothesis is less likely to account for event E than the alternative hypothesis of intelligent design [7]. The CSI of event E is the penalized specificity,

$$\chi = -\log_2[10^{120} \cdot \varphi_S(T) \cdot \mathbf{P}(T \mid \mathbf{H})] \approx \sigma - 399 \text{ bits}.$$

Intelligent design is inferred for event $\mathbf{H}$ if $\chi > 1$ or, equivalently, specificity in excess of 400 bits [7]. Note that Dembski sometimes invokes the argument from specified complexity to reject chance in favor of human intelligence, and in these cases he sets the number of replicational resources smaller [7].[5]

[5] ID advocates hold that human intelligence is not natural (materialistic). Thus humans can cause events with high levels of CSI.

5.2 Some Criticisms of Specified Complexity

It seems that Dembski, as a mathematician and philosopher, thinks more analytically than algorithmically. Most of the following addresses aspects of computation of CSI. It is important to keep in mind the adversarial aspect of the argument from specified complexity. Chance hypotheses should come from mainstream scientists, not ID advocates. They often will be analytically intractable, and design inferences will require direct computation of CSI. Below are listed some major criticisms of Dembski's arguments.

A model of nature is conflated with nature itself.

Recall that the chance hypothesis is essentially that natural causes account entirely for a natural event. The design inference is a claim that natural causes alone do not suffice to explain the event. But in practice the chance hypothesis that is likely to be derived from a scientific model, and what is subject to rejection is not natural causation itself, but the model of natural causation. The distinction is of vital importance. If scientists do not understand some class of events, a chance hypothesis derived from their best model may be rejected in favor of design. The inability of the model to account for the event is treated as the inability of natural causation to account for the event. This constitutes a logically fallacious argument from ignorance. And as described above, ID advocates indeed focus on biological entities with histories that are very difficult to determine.

Key aspects of CSI are not explicit in Dembski's treatment.

In conventional mathematical terms, a "pattern" described by a semiotic agent is a *property*. A property T is a subset of some set U, and saying that $x \in U$ *has property* T is equivalent to saying that $x \in T$. Let D_S denote the set of all descriptions that semiotic agent S may emit. For all descriptions d in D_S, let $\varphi_S(d)$ be the rank-complexity of d described above. Let $D_S(E) \subseteq D_S$ be the set of all descriptions associated with event E by S. Finally, let

$$T_S(d) = \{\omega \in \Omega \mid \omega \text{ has the property } S \text{ describes with } d\},$$

where Ω is the sample space. This glosses over semantic interpretation of the descriptions in D_S. Nonetheless, it should convey that there is no way to determine the rejection region without knowing both its description and the semantics of the semiotic agent that generated the description. Then for all semiotic agents S and for all descriptions d in $D_S(E)$ the CSI is

$$\chi_S(d) = -\log_2[10^{120} \cdot \varphi_S(d) \cdot \mathbf{P}(T_S(d) \mid \mathbf{H})].$$

This appropriately indicates that CSI is associated with descriptions of event E. For completeness, one may define $\chi(E)$ as the maximum of $\chi_S(d)$ over all S and d, but maximization is infeasible in practice, and design inference requires only $\chi_S(d) > 1$ for some S and d.

"Divide-and-conquer" rejection of disjunctive hypotheses is permitted.

When there are multiple chance hypotheses $\{\mathbf{H}_i\}$, they must be rejected jointly to infer intelligent design. Dembski fails to point out that the semiotic agent S and the description d in $D_S(E)$ must be held constant while rejecting all hypotheses [7]. This requirement is captured by generalizing the definition of χ_S to

$$\chi_S(d) = -\log_2[10^{120} \cdot \varphi_S(d) \cdot \max_i \mathbf{P}(T_S(d) \mid \mathbf{H}_i)].$$

CSI is not computable.

For Dembski, the physical (material) universe is discrete and finite, and so is Ω [3, 4, 7]. This would seem to bode well for computation of CSI, but problems arise from the fact that a semiotic agent may associate with event E a description of a property defined on an infinite set. Many finitely describable properties are not algorithmically decidable [13], irrespective of the nonexistence of infinite sets in the physical universe.

The value of $\mathbf{P}(T_S(d) \mid \mathbf{H})$ is the sum of $\mathbf{P}(\omega \mid \mathbf{H})$ over all points ω in rejection region $T_S(d)$. Its computation generally requires conversion of description d into an algorithm that decides which points in Ω have the described property. But if the described property is not decidable, $\mathbf{P}(T_S(d) \mid \mathbf{H})$ is computable only under special circumstances. This holds even if the initial "translation" of d into an algorithm is non-algorithmic.

Incomputable properties are especially likely to arise in the important case that Ω is a set of entities that describe or compute partial (not always total) recursive functions. An example is the set of all LISP programs of length not exceeding some large bound. A semiotic agent's description of program E will commonly refer to a nontrivial property of the function computed by E. But a key result in the theory of computation, Rice's theorem, implies that no algorithm decides whether other LISP programs compute functions with that property [13]. In other words, there is generally no algorithm to say whether programs in Ω belong to the rejection region. This indicates that for a wide range of computational entities CSI may be computed only for the form (e.g., the source code), and not the function. Note that some philosophers and scientists believe that brains compute partial (not total) recursive functions [8].

Some design hypotheses call for nonexistent chance hypotheses.

In conventional statistical hypothesis testing, one begins with an alternative hypothesis and then selects a chance hypothesis. This does not carry over to the argument from specified complexity. An ID advocate may believe an event is designed, but mainstream scientists may not have provided an appropriate

chance hypothesis to reject. The non-existence of the hypothesis (scientific model) may be due to scientific indifference or scientific ignorance.

As an example of scientific indifference, consider what is required to compute the CSI of the bacterial flagellum, which Dembski *qua* semiotic agent describes as a "bidirectional rotary motor-driven propeller" [7]. The sample space contains biological structures detached from whole phenotypes, and the chance hypothesis must associate probabilities of evolution with them. But nothing in evolutionary theory leads to such a hypothesis, and it is absurd to insist that scientists to supply one.

Ignorance is ubiquitous in science, and some phenomena (e.g., gravity) have resisted explanation for centuries. The inability of science to explain a class of events does not constitute the least evidence for ID. To suggest otherwise is to engage in a logical fallacy known as argument from ignorance.

Computation of CSI may be infeasible when theoretically possible.

If Ω is the set of all biological structures (begging the question of how to define "biological structure") that have existed (begging the question of how to determine all structures of entities that have ever lived) or might have existed (begging the question of how to determine what might have lived), how will an algorithm efficiently locate the points in the sample space with the property "bidirectional rotary motor-driven propeller"? No approach other than exhaustive exploration of the sample space for points with the property is evident. The time required for such a computation makes it infeasible. Furthermore, the practicality of defining the sample space for an algorithm to operate upon is highly dubious.

Another feasibility issue is the cost of computing $\mathbf{P}(\omega \mid \mathbf{H})$ for a single ω in Ω. Suppose $\mathbf{P}(\omega \mid \mathbf{H})$ is, loosely speaking, the probability of evolution of ω, and that $\mathbf{H}$ is derived from a simulation model supplied by a scientist. The results of a simulation run usually depend upon initial conditions and parameter settings. There will virtually always be uncertainty as to how to set these values, and the consequence is that many runs of the simulation model (with various settings) will be required to obtain $\mathbf{P}(\omega \mid \mathbf{H})$.

Putative innovations in statistical hypothesis testing have not passed peer review.

Dembski's approach to design inference [7] is correct only if he has made monumental contributions to statistical hypothesis testing. There is nothing precluding publication of his statistical work in a peer-reviewed journal of mathematics or statistics. At the time of this writing, Dembski has not published any of his work. Consequently, one must regard his statistical reasoning with skepticism.

5.3 The Law of Conservation of Information

In earlier work [4], Dembski argues informally for a *law of conservation of information*, which does not specify that complex specified information is strictly conserved in natural processes, but that gain of CSI is bounded above by 500 bits. That is, a closed physical system may go from a state of lower CSI to a state of higher CSI, but the increase cannot exceed 500 bits. The bound corresponds to a putative limit on the improbability of events in the physical universe, as described below. Dembski regards evolutionary computations (ECs) as closed systems, and if an EC produces an apparent gain of more than 500 bits of CSI in its population, he argues that humans have surreptitiously (perhaps haplessly) added CSI to the process [4].

The 500-bit bound on CSI gain is the negative logarithm of the *universal probability bound* Dembski advocates in earlier work, 10^{-150} [3, 4]. He considers events with probability below the bound to be effectively impossible. Dembski [3] cites Émile Borel, who is quoted in the epigraph of this chapter, as a famous proponent of a universal probability bound. In fact Borel selects different bounds for different applications – they are hardly "universal" [2]. Some are much smaller, and some much larger, than Dembski's bound. In the work detailed above, Dembski indicates that "instead of a static universal probability bound of 10^{-150} we now have a dynamic one of" $10^{-120}/\varphi_S(d)$ [7]. That is, the bound is adapted to the observer of an event and the observer's description of the event. This is in marked contrast with Borel's approach.

Dembski does not indicate in [7] how to rescue the law of "conservation" of information. He states, however, that $\varphi_S(d)$ should not exceed 10^{30} in practice, and observes that his old static bound of 10^{-150} is a lower bound on the dynamic bound. This suggests that Dembski may renew his claim that CSI gain cannot exceed 500 bits in a natural process. With the dependence of CSI upon observers and their descriptions of events, what it means to gain CSI is hardly obvious.

6 ID and Evolutionary Computation

[This section is omitted.]

7 Conclusion

We have criticized ID theory for its intrinsic faults. But in the end the only way to understand the theory is as a veiled apologetics. Jews, Christians, and Muslims agree that the God of Abraham created the diverse forms of life on earth, imbuing only humans with a capacity to create ex nihilo. Although some of the faithful accept that religion and science are different belief systems leading to different beliefs, others insist that science must never contradict

religion. ID theorists begin with religious beliefs about life and humanity, and attempt to show that contradictory beliefs held by almost all mainstream scientists are wrong. They hide their religious motivation because they hope their theory will find its way into science classes of public schools.

Irreducible complexity is the weakest part of the apologetics. Behe has had to concede what Muller pointed out decades before he was born, namely that indirect evolutionary pathways may give rise to irreducible complexity. And there is good fossil evidence that the interconnected bones of the mammalian middle ear evolved from a reptilian jawbone. The Avida simulation is reasonably interpreted as generating irreducibly complex programs. ID advocates continue, however, to focus on irreducibly complex biosystems for which there are few historical data (e.g., the flagellum). They argue that evolutionary theory fails to account for the emergence of these systems when in fact there are few hard data.

The argument from specified complexity rests on an approach to statistical hypothesis testing that has not passed peer review. Even if the statistical foundation is sound, the argument is logically flawed. When it claims to reject purely natural causation in favor of design, it actually rejects a model. That is, if there is no good model of a phenomenon, then the argument from specified complexity reduces to argument from ignorance. Even with an excellent model, specified complexity is in some cases impractical to compute, or even incomputable.

Dembski's claim that all entities with high specified complexity are intelligently designed seems to have been falsified by various evolutionary computations. But Dembski argues constantly that experimenters have smuggled intelligence into the computations. Accumulating further computational evidence should be valuable, but in the end formal mathematical analysis may be required to settle the dispute.

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